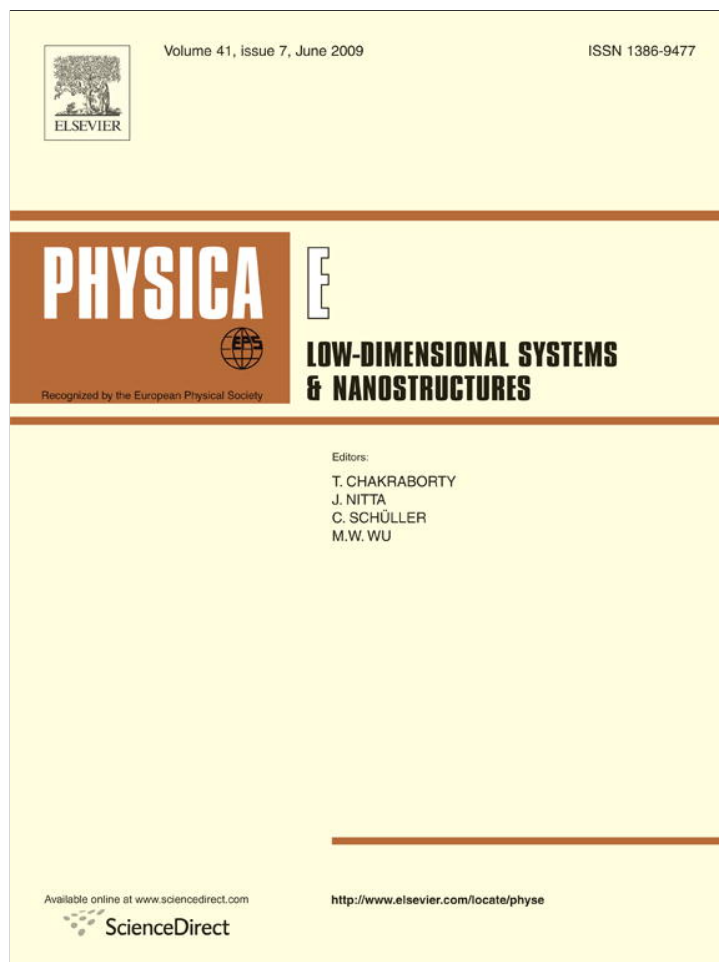


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# Spin-dependent transmission with Rashba interaction through a quantum wire with a side-coupled quantum-dot array

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## ABSTRACT

The non-equilibrium Green's function method is applied to the system formed by a quantum-dot (QD) array side coupled to a one-dimensional quantum wire (QW), which is attached to normal leads. The system is modelled by a single-band "tight-binding" Hamiltonian with Rashba spin-orbit interaction. Using the recursive Green function method, independently of the length of the QW and the QD array, the system is reduced to a three equivalent sites with the effect of the QD array included in the central site, and the couplings to leads in the two extreme sites. Then, the transmission is studied in two cases: Firstly, when the QD array is of the same material as that of the QW and it is not magnetic, there is no preferential spin direction, and the Fano resonances and antiresonances are analyzed as a function of the Rashba parameter and local energies in the QD array. Secondly, when the QD array is assumed to be a metallic ferromagnet, the energy of the up and down states is different and a spin-dependent transmission is obtained.

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## 1. Introduction

The study of spin-dependent transmission is a main area of research in mesoscopic devices, where interaction and interference phenomena define the properties of quantum transport. On the one hand, the Rashba spin-orbit interaction (RSOI) plays a fundamental role as a scattering mechanism between the up ( $\uparrow$ ) and down ( $\downarrow$ ) spin states of the electron in a quantification axis. Many studies along these lines, both theoretical [1–9] and experimental [10,11], have been carried out. On the other hand, interference effects between a bound state immersed in a continuous band and the continuum have been used to produce Fano resonances (perfect transmission) and antiresonances (zeros of transmission) in the conductance of a quasi-one-dimensional (Q1D) electron guide [12]; of a quantum wire (QW) with a side-coupled quantum-dot (QD) array [13]; of a double QD molecule attached to leads [14]; and also, the Fano effect has been experimentally observed in a QD side-coupled to QW [15]. A first proposal of a spin filter [16] uses a one-dimensional ring subject to a magnetic field that originate a Zeeman splitting which results in different adiabatic transmission components,  $T_{\downarrow\uparrow} > T_{\uparrow\downarrow}$ ; while the nonadiabatic components are the same,  $T_{\uparrow\uparrow} = T_{\downarrow\downarrow}$ . However, in [12–16] the RSOI was not taken into account. Kiselev and Kim

[17] show how to get spin-polarized fluxes in three (at least) terminal devices, only considering Rashba spin-orbit coupling. Later [6,7], it is shown how the RSOI can create Fano lineshapes in the transmission of a quasi-one-dimensional quantum wire. Therefore, various resonant structures with RSOI have been analyzed, like a Rashba QD subject to a magnetic field and coupled to ferromagnetic leads [18]; a T-shaped double QD [19]; a T-shaped waveguide [20,21] and spin filters based on quantum rings [22,23].

In this work, first, a quasi-one-dimensional system formed by two conductors that are coupled, is analyzed; and the Green's functions of the whole system are expressed in terms of those of the isolated conductors and the coupling between them. Then, these results are applied to a one-dimensional system formed by a QD array side-coupled to a QW considering RSOI and with the QW connected to normal leads. The non-equilibrium Green's function method [24,25] is used to compute the spin-dependent transmissions  $T_{\sigma\sigma'}$ ,  $\sigma, \sigma' = \uparrow, \downarrow$ , at zero temperature. If we do not consider electron–electron interaction, as in [8,13,18,21], we see that every site in the system can be described by a  $2 \times 2$  Green's function matrix. Then, the system is reduced to three generalized (equivalent) sites which take into account the whole system and its coupling to leads. When the QD array is of the same material as the QW there exists time-reversal symmetry, so there is no preferential spin, and the results can be seen as an extension to [13,26]. When the QD array is assumed to be a magnetic material with energy levels at each dot  $\varepsilon_{i0\uparrow} = -\varepsilon_{i0\downarrow} = -\varepsilon_{i0}$ , we get  $T_{\uparrow\uparrow} \neq T_{\downarrow\downarrow}$  and  $T_{\uparrow\downarrow} = T_{\downarrow\uparrow}$ .

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## 2. Theory

### 2.1. Quasi-one-dimensional system

We consider the isolated conductors C1 and C2 of dimensions  $nx1$  sites in  $x$  axis and  $nc1$  in  $y$  axis for C1, and similarly  $nx2$ ,  $nc2$  for C2. Each one is described by its Q1D single-band tight-binding Hamiltonian including RSOI,  $H_1$  and  $H_2$ , respectively; and let  $H_{12}$  be, the interaction between them when the site column at  $i = nx1$  of C1 is coupled to the site column  $i = 1$  of C2. Then, the Hamiltonian of the total system  $H$ , will be [2]

$$H = H_1 + H_2 + H_{12} \quad (1a)$$

$$H_{12} = t_x^{ac1} \sum_{k=1}^{nc1} [C_{1k\sigma}^{C2,+} C_{nx1,k\sigma}^{C1} + h.c.] - t_{SO}^{ac1} \sum_{k=1}^{nc1} [C_{nx1,k\uparrow}^{C1,+} C_{1k\downarrow}^{C2} - C_{nx1,k\downarrow}^{C1,+} C_{1k\uparrow}^{C2} + h.c.] \quad (1b)$$

where  $C_{ik\sigma}^{C1,+}$ ,  $C_{ik\sigma}^{C1,-}$ ,  $C_{ik\sigma}^{C2,+}$  and  $C_{ik\sigma}^{C2,-}$ ,  $1 \leq i \leq nx1, nx2$ ,  $1 \leq k \leq nc1, nc2$ ,  $\sigma = \uparrow, \downarrow$ , with  $nc1 \leq nc2$ ; are the creation and destruction operators in C1 and C2, respectively. The equation of motion method is applied to the retarded Green's functions of  $H$  [25]. They can be grouped in:  $G_{ij\sigma, i'j'\sigma'}^R(t-t')$ , with the two sites in C1; the first site in C1 and the second one in C2,  $G_{i1j\sigma, i2j'\sigma'}^R(t-t')$ ; the first site in C2 and the second one in C1,  $G_{i2j\sigma, i1j'\sigma'}^R(t-t')$  and;  $G_{i2j\sigma, i2j'\sigma'}^R(t-t')$ , with the two sites in C2. Then, making the Fourier transform, we arrive at

$$(G_{C1-C1}^{R0})^{-1} G_{C1,n1}^R = I_{n1} + A1 G_{C2,n1}^R \quad (2a)$$

$$(G_{C2-C2}^{R0})^{-1} G_{C2,n1}^R = A1^+ G_{C1,n1}^R \quad (2b)$$

where  $G_{C1-C1}^{R0}$  and  $G_{C2-C2}^{R0}$  are the square matrix of Green's functions of the isolated C1 and C2, respectively;  $G_{C1,n1}^R$  and  $G_{C2,n1}^R$  the column vector of Green's functions of the complete system, with  $n1$  belonging to C1;  $I_{n1}$  the column vector with a 1 at row  $n1$  and 0 in the remaining rows and;  $A1$  the coupling matrix between C1 and C2 (dimension equal to  $2 \cdot nx1 \cdot nc1 \bullet 2 \cdot nx2 \cdot nc2$ ). Similar equations to (2a) and (2b) can be written with  $C1 \rightarrow C2$ ,  $C2 \rightarrow C1$ ,  $n1 \rightarrow n2$  and  $A1 \rightarrow A1^+$ . Eqs. (2a) and (2b) can be shown as the recursive Green's function method [9,27,28], applied to our problem.

### 2.2. One-dimensional system with RSOI

Now, we consider the one-dimensional system with RSOI shown in Fig. 1, which is formed by a quantum wire (initially a three-site

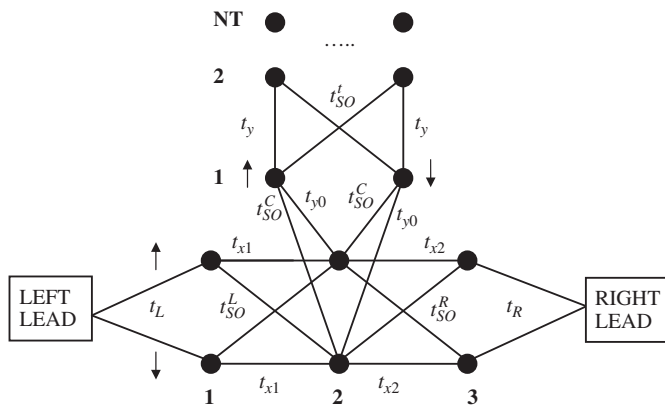


Fig. 1. A quantum-dot array side coupled to a quantum wire decomposed in its states up ( $\uparrow$ ) and down ( $\downarrow$ ), showing the RSOI.

chain) with a side-coupled quantum-dot array (transversal chain, TC) attached to the QW [7,13]. The Hamiltonian is  $H = H_L + H_{L1} + H_{cen} + H_{CTC} + H_{TC} + H_{3R} + H_R$ , where  $H_L$  and  $H_R$  are the Hamiltonians of the left (L) and right (R) lead;  $H_{L1}$  is the coupling between L and the site 1 of the QW;  $H_{cen}$  is the Hamiltonian of the isolated QW;  $H_{CTC}$  is the coupling between the site 2 of the QW and the site 1 of the TC;  $H_{TC}$  is the Hamiltonian of the isolated QD array of length  $NT$  dots; and  $H_{3R}$  is the coupling between the site 3 of the QW and R. So we have

$$H_{L1} = \sum_{k \in L, \sigma = \uparrow, \downarrow} (t_L C_{k\sigma}^+ C_{1\sigma} + h.c.) \quad (3a)$$

$$H_{cen} = \sum_{l=1, \sigma}^3 \varepsilon_{l\sigma} C_{l\sigma}^+ C_{l\sigma} + \sum_{\sigma = \uparrow, \downarrow} (t_{x1} C_{1\sigma}^+ C_{2\sigma} + t_{x2} C_{2\sigma}^+ C_{3\sigma} + h.c.) - t_{SO}^L (C_{1\uparrow}^+ C_{2\downarrow} + C_{2\downarrow}^+ C_{1\uparrow} - C_{1\downarrow}^+ C_{2\uparrow} - C_{2\uparrow}^+ C_{1\downarrow}) - t_{SO}^R (C_{2\uparrow}^+ C_{3\downarrow} + C_{3\downarrow}^+ C_{2\uparrow} - C_{2\downarrow}^+ C_{3\uparrow} - C_{3\uparrow}^+ C_{2\downarrow}) \quad (3b)$$

$$H_{CTC} = \sum_{\sigma = \uparrow, \downarrow} (t_{y0} C_{2\sigma}^+ C_{t1\sigma} + h.c.) + it_{SO}^C (C_{2\uparrow}^+ C_{t1\downarrow} - C_{t1\downarrow}^+ C_{2\uparrow} + C_{2\downarrow}^+ C_{t1\uparrow} - C_{t1\uparrow}^+ C_{2\downarrow}) \quad (3c)$$

$$H_{TC} = \sum_{m=1, \sigma}^{NT} \varepsilon_{tm\sigma} C_{tm\sigma}^+ C_{tm\sigma} + \sum_{m, \sigma}^{NT-1} (t_y C_{tm\sigma}^+ C_{tm+1, \sigma} + h.c.) + it_{SO}^T \sum_m^{NT-1} (C_{tm\uparrow}^+ C_{tm+1\downarrow} + C_{tm\downarrow}^+ C_{tm+1\uparrow} - C_{tm+1\downarrow}^+ C_{tm\uparrow} - C_{tm+1\uparrow}^+ C_{tm\downarrow}) \quad (3d)$$

$$H_{3R} = \sum_{k \in R, \sigma = \uparrow, \downarrow} (t_R C_{k\sigma}^+ C_{3\sigma} + h.c.) \quad (3e)$$

The quantum-dot array can be considered as if the first QD would be coupled to the remaining chain, the latter being described by a  $2 \times 2$  Green's function matrix. That remaining chain can be considered as the generalized transversal site label as 2 [26]; so, denoting such Green's functions by  $G_{t2\uparrow, t2\uparrow}^{R, S2}$ ,  $G_{t2\uparrow, t2\downarrow}^{R, S2}$  and  $G_{t2\downarrow, t2\downarrow}^{R, S2}$ , and using Eqs. (2a) and (2b) with  $n1 = t1 \uparrow$  or  $t1 \downarrow$ , it follows that:

$$\begin{pmatrix} \varepsilon - \varepsilon_{t1\uparrow} & 0 \\ 0 & \varepsilon - \varepsilon_{t1\downarrow} \end{pmatrix} \begin{pmatrix} G_{t1\uparrow, n1}^{R, S1} \\ G_{t1\downarrow, n1}^{R, S1} \end{pmatrix} = I_{n1} + \begin{pmatrix} t_y & it_{SO}^t \\ it_{SO}^t & t_y \end{pmatrix} \times \begin{pmatrix} G_{t2\uparrow, t2\uparrow}^{R, S2} & G_{t2\uparrow, t2\downarrow}^{R, S2} \\ G_{t2\downarrow, t2\uparrow}^{R, S2} & G_{t2\downarrow, t2\downarrow}^{R, S2} \end{pmatrix} \begin{pmatrix} t_y & -it_{SO}^t \\ -it_{SO}^t & t_y \end{pmatrix} \begin{pmatrix} G_{t1\uparrow, n1}^{R, S1} \\ G_{t1\downarrow, n1}^{R, S1} \end{pmatrix} \quad (4)$$

In fact, the three sites in the QW can be considered the sites  $(nx1-1)$ ,  $nx1$  and  $(nx1+1)$  of a larger wire, with  $t_{nx1-2}$  the hopping between the states at  $(nx1-2)$  and  $(nx1-1)$  of the same spin,  $t_{x1} \rightarrow t_{nx1-1}$  and  $t_{x2} \rightarrow t_{nx1}$  and;  $t_{SO}^L$  and  $t_{SO}^R$  would represent the RSOI in the QW to the left and right of  $nx1$ , respectively. Now, we consider the subsystem to the left of  $nx1-1$ , which is formed by: all the sites in the QW with  $1 \leq l \leq nx1-2$ , the left lead and the coupling between them. Such a subsystem is described by the  $2 \times 2$  Green's function matrix  $G_{(nx1-2, \sigma)}^{R, IS}$   $\sigma = \uparrow, \downarrow$ ; so, it can be written as

$$\begin{pmatrix} g_{nx1-1\uparrow, nx1-1\uparrow}^R & g_{nx1-1\uparrow, nx1-1\downarrow}^R \\ g_{nx1-1\downarrow, nx1-1\uparrow}^R & g_{nx1-1\downarrow, nx1-1\downarrow}^R \end{pmatrix}^{-1} \begin{pmatrix} G_{nx1-1\uparrow, nx1-1\uparrow}^{R, IS} & G_{nx1-1\uparrow, nx1-1\downarrow}^{R, IS} \\ G_{nx1-1\downarrow, nx1-1\uparrow}^{R, IS} & G_{nx1-1\downarrow, nx1-1\downarrow}^{R, IS} \end{pmatrix} = I + \begin{pmatrix} t_{nx1-2}^* & t_{SO}^L \\ -t_{SO}^L & t_{nx1-2}^* \end{pmatrix} G_{(nx1-2, \sigma)}^{R, IS} \begin{pmatrix} t_{nx1-2} & -t_{SO}^L \\ t_{SO}^L & t_{nx1-2} \end{pmatrix} \times \begin{pmatrix} G_{nx1-1\uparrow, nx1-1\uparrow}^{R, IS} & G_{nx1-1\uparrow, nx1-1\downarrow}^{R, IS} \\ G_{nx1-1\downarrow, nx1-1\uparrow}^{R, IS} & G_{nx1-1\downarrow, nx1-1\downarrow}^{R, IS} \end{pmatrix} \quad (5)$$

where  $g_{(nx1-1,\sigma)}^R$ , is the Green's function matrix of the isolated site  $nx1-1$ , with  $(g_{nx1-1,\sigma,nx1-1\sigma}^R)^{-1} = \varepsilon - \varepsilon_{nx1-1\sigma}$ ,  $\sigma = \uparrow\downarrow$  and  $g_{nx1-1\uparrow,nx1-1\downarrow}^R = g_{nx1-1\downarrow,nx1-1\uparrow}^R = 0$ ;  $G_{(nx1-1,\sigma)}^{R,IS}$  the Green's function matrix of the generalized site  $nx1-1$ , decoupled from the generalized sites  $nx1$  and  $nx1+1$ . A similar equation to (5) can be written for the generalized site  $nx1$ , which would include the effect of the transversal chain. Also, for the generalized site  $nx1+1$  an equation like (5), which includes the right lead and its coupling to the QW, is obtained. Now, considering the coupling between these three equivalent sites, the following equation for the retarded Green's functions of the complete system, can be reached:

$$\begin{pmatrix} [G_{(nx1-1,\sigma)}^{R,IS}]^{-1} & -t_{nx1-1}^L & t_{SO}^L & 0 & 0 \\ & -t_{SO}^L & -t_{nx1-1} & 0 & 0 \\ -t_{nx1-1}^* & -t_{SO}^L & [G_{(nx1,\sigma)}^{R,IS}]^{-1} & -t_{nx1} & t_{SO}^R \\ t_{SO}^L & -t_{nx1-1}^* & & -t_{SO}^R & t_{nx1} \\ 0 & 0 & -t_{nx1}^* & -t_{SO}^R & \\ 0 & 0 & t_{SO}^R & -t_{nx1}^* & [G_{(nx1+1,\sigma)}^{R,IS}]^{-1} \end{pmatrix} \times \begin{pmatrix} G_{nx1-1\uparrow,n}^R \\ G_{nx1-1\downarrow,n}^R \\ G_{nx1\uparrow,n}^R \\ G_{nx1\downarrow,n}^R \\ G_{nx1+1\uparrow,n}^R \\ G_{nx1+1\downarrow,n}^R \end{pmatrix} = I_n \quad (6)$$

with  $n$  equal to anyone of the six possible states. Eq. (6) is a Dyson equation of the system described by the Hamiltonian  $H$ , and can be written like  $(g^{-1} - \Sigma^R) \cdot G^R = I$ ,  $\Sigma^R$  being the retarded self-energy.

The level-width function is given by

$$\Gamma = i[\Sigma^R - (\Sigma^R)^*] = \begin{pmatrix} \Gamma_{\uparrow\uparrow}^L & \Gamma_{\uparrow\downarrow}^L & 0 & 0 & 0 & 0 \\ \Gamma_{\downarrow\uparrow}^L & \Gamma_{\downarrow\downarrow}^L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Gamma_{\uparrow\uparrow}^R & \Gamma_{\uparrow\downarrow}^R \\ 0 & 0 & 0 & 0 & \Gamma_{\downarrow\uparrow}^R & \Gamma_{\downarrow\downarrow}^R \end{pmatrix} = \Gamma^L + \Gamma^R \quad (7)$$

In general,  $\Gamma_{\sigma\sigma'}^{L,R}$ ,  $\sigma, \sigma' = \uparrow\downarrow$  will be different of zero, since we are dealing with the subsystems  $(nx1-1)$  and  $(nx1+1)$ ; which take into account the single site and all that exits to the left and right of that single site; in particular, the RSOI along the QW and the coupling to the left and right leads. At zero temperature, using the formula for the transmission [24,25]  $T = Tr\{\Gamma^L G^R \Gamma^R G^A\}$  and; considering  $\Gamma^L(\Gamma^R)$  as the  $2 \times 2$  matrix resulting from keeping only the first (last) two rows and columns of Eq. (7); the following expressions are obtained:

$$T = Tr\{\Gamma^L G_{(nx1-1,\sigma,nx1+1\sigma')}^R \Gamma^R G_{(nx1+1\sigma',nx1-1\sigma)}^A\} = T_{\uparrow\uparrow} + T_{\uparrow\downarrow} + T_{\downarrow\uparrow} + T_{\downarrow\downarrow} \quad (8a)$$

$$T_{\sigma\sigma} = \left( \sum_{\sigma'\sigma''=\uparrow\downarrow} \Gamma_{\sigma\sigma'}^L G_{nx1-1,\sigma'nx1+1,\sigma''}^R \Gamma_{\sigma''\sigma}^R \right) (G_{nx1-1,\sigma nx1+1\sigma}^R)^* \quad (8b)$$

$$T_{\sigma\bar{\sigma}} = \left( \sum_{\sigma'\sigma''=\uparrow\downarrow} \Gamma_{\sigma\sigma'}^L G_{nx1-1,\sigma'nx1+1,\sigma''}^R \Gamma_{\sigma''\bar{\sigma}}^R \right) (G_{nx1-1,\sigma nx1+1,\bar{\sigma}}^R)^* \quad (8c)$$

It is clear that in  $T_{\uparrow\uparrow}$  there will be contributions from paths that begin in an up state at the left lead; due to the RSOI, that path

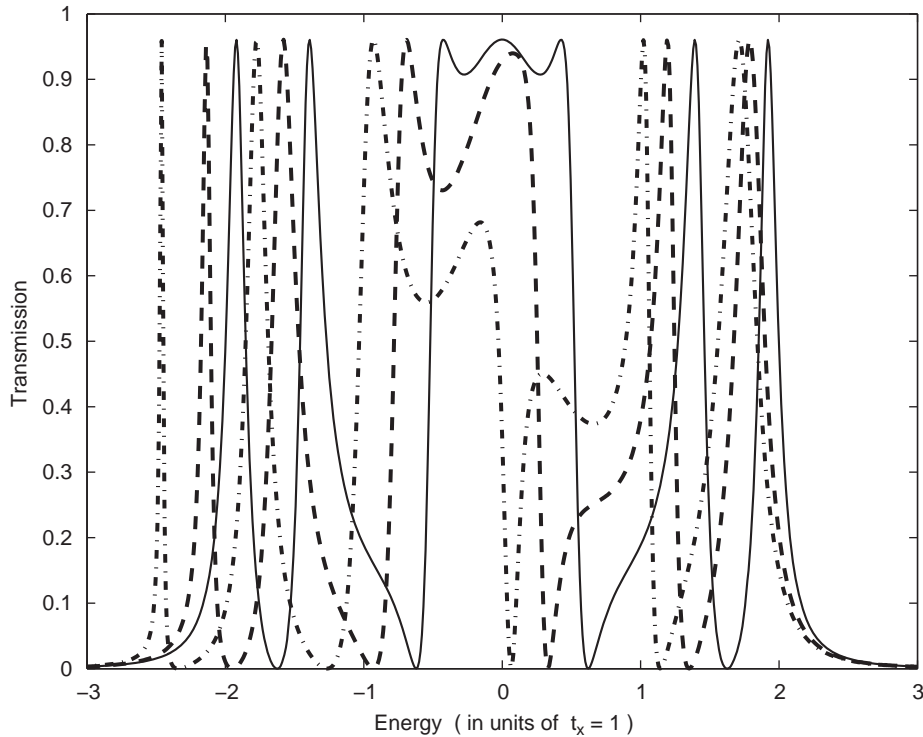


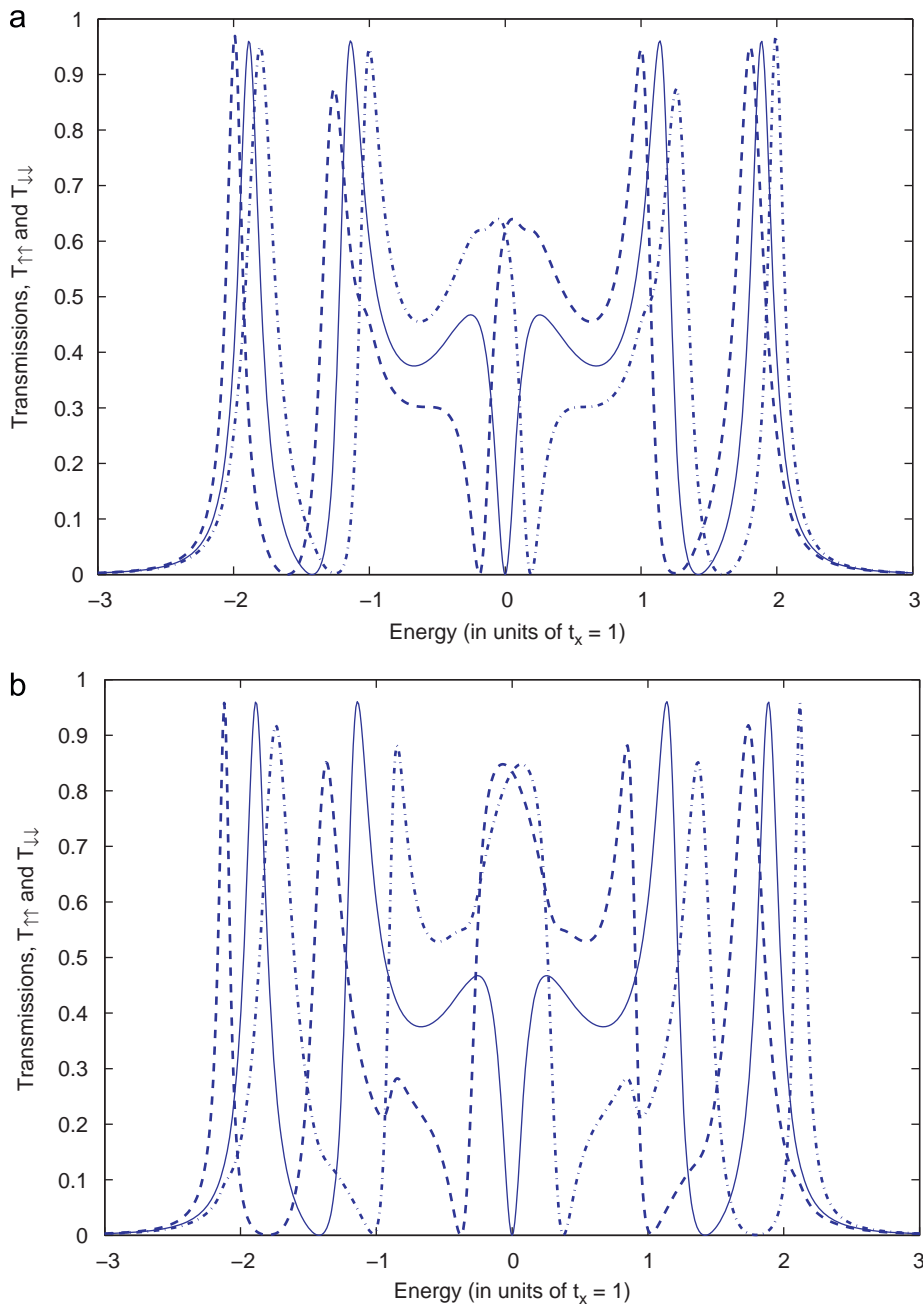
Fig. 2. Transmission  $T_{\uparrow\uparrow} (= T_{\downarrow\downarrow})$  with  $NT = 4$  and for different value pairs of  $\varepsilon_{t0}$  and  $t_{SO}^t \cdot t_{SO}^t = 0.1$ ,  $\varepsilon_{t0} = 0.0$ , solid line;  $t_{SO}^t = 0.2$ ,  $\varepsilon_{t0} = -0.3$ , dashed line and  $t_{SO}^t = 0.4$ ,  $\varepsilon_{t0} = -0.6$  dash-dot line.

passes through down states along the QW and reaches the state  $(n\uparrow 1 - 1, \uparrow)$ .

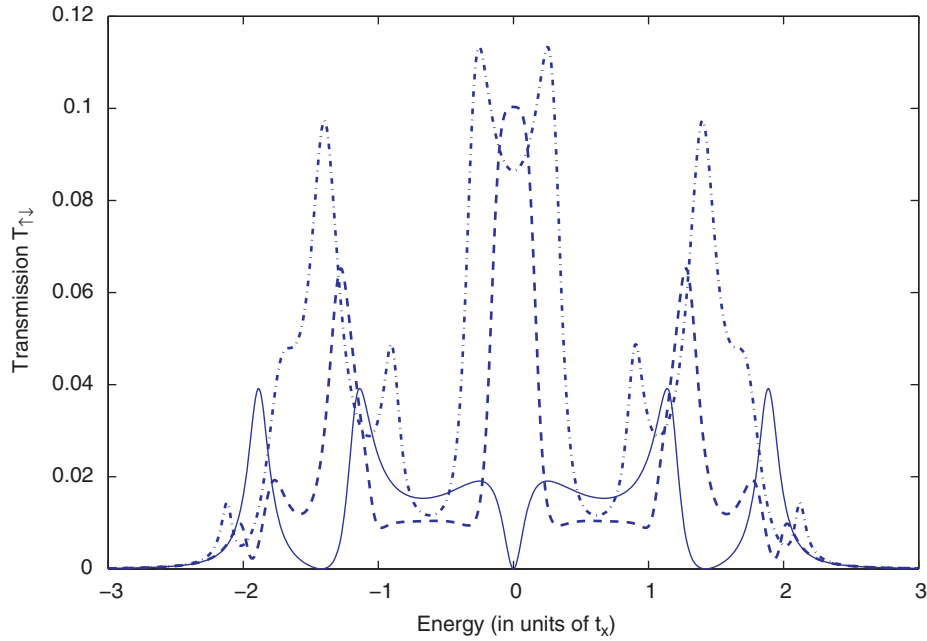
### 3. Results and discussion

The numerical results can be classified in two groups depending on whether the material of the QW is the same as that one of the TC or not. The coupling to the leads is specified by a retarded self-energy  $\Sigma_{L,R}^R = \Lambda_{L,R} + i\Gamma_{L,R}/2$ , independent of the energy as in [26], with  $\Lambda_{L,R} = 0$  and  $\Gamma_{L,R} = 1.0$ . When the materials are the same, no magnetic and uniform,  $t_x = t_{y0} = t_y = t$ ,  $t_{SO}^L = t_{SO}^C = t_{SO}^R = t_{SO}^W = t_{SO}^t$  and, by time-reversal symmetry  $T_{\uparrow\uparrow} = T_{\downarrow\downarrow}$  and  $T_{\uparrow\downarrow} = T_{\downarrow\uparrow}$ . Being  $\varepsilon_{l\sigma}$  of the QW and  $\varepsilon_{tm\sigma}$  of the TC equal to zero, the centre of the energy band is zero. Then, it is found that

when  $NX = 3$  (odd) and  $NT$  is odd,  $T_{\uparrow\uparrow} (= T_{\downarrow\downarrow}) = 0$  at  $\varepsilon = 0$ ; whereas if  $NT$  is even, because of the RSOI,  $T_{\uparrow\uparrow}(\varepsilon = 0)$  is slightly less than 1, but  $T_{\uparrow\uparrow} + T_{\downarrow\downarrow} = 1$ . With  $NX$  even and  $NT$  odd, we have also  $T_{\uparrow\uparrow}(\varepsilon = 0) = 0$ ; but now, when  $NT$  is even  $T_{\uparrow\uparrow} + T_{\downarrow\downarrow}$  is notably less than 1 (no perfect transmission). Since we take into account the length of the QW, its coupling to the leads and the RSOI, these results are an extension of those in [13,26]. From now on, a quantum wire with only three sites is considered, so that  $\Gamma_{\uparrow\downarrow}^{L,R} = \Gamma_{\downarrow\uparrow}^{L,R} = 0$ ; and Eq. (6) is solved to compute the  $2 \times 2$  matrix  $G_{(1\sigma,3\sigma)^R}$ . Applying a gate voltage ( $V_g$ ) only to the transversal chain, the energies  $\varepsilon_{tm\sigma}$  of the quantum dots and the Rashba parameter  $t_{SO}^t$  modify simultaneously. Fig. 2 shows  $T_{\uparrow\uparrow} (= T_{\downarrow\downarrow})$  as a function of the energy with  $t = 1.0$ ,  $\varepsilon_{l\sigma} = 0$ ,  $t_{SO}^W = 0.1$ ,  $NT = 4$  and various value pairs of  $\varepsilon_{tm\sigma} = \varepsilon_{t0}$  (for any  $m$  and  $\sigma$ ) and  $t_{SO}^t$ ; corresponding to a given value of  $V_g$ . We have chosen arbitrarily,  $t_{SO}^t = 0.1$ ,



**Fig. 3.** Transmissions  $T_{\uparrow\uparrow}$  (dashed line) and  $T_{\downarrow\downarrow}$  (dash-dot line) when the transversal chain is a magnetic material with  $\varepsilon_{t0\uparrow} = -\varepsilon_{t0\downarrow} = -\varepsilon_{t0} = -0.2$  (a), and  $\varepsilon_{t0} = 0.4$  (b). For reference, the curve with  $\varepsilon_{t0} = 0$  is also shown (solid line).



**Fig. 4.**  $T_{\uparrow\downarrow} (= T_{\downarrow\uparrow})$  for different values of  $\epsilon_{t0}$ :  $\epsilon_{t0} = 0$ , solid line;  $\epsilon_{t0} = 0.2$ , dashed line and;  $\epsilon_{t0} = 0.4$ , dash-dot line.

$\epsilon_{t0} = 0.0$ ;  $t_{SO}^t = 0.2$ ,  $\epsilon_{t0} = -0.3$  and  $t_{SO}^t = 0.4$ ,  $\epsilon_{t0} = -0.6$ . It can be shown that the zeros of the transmission are at  $\epsilon_{i0} = \pm\sqrt{(3 \pm \sqrt{5})/2} \cdot |t_y|$ ,  $i = 1, 2, 3$  and  $4$ ; if we do not consider the RSOI in the transversal chain and without  $V_g$  applied. These antiresonances occur whenever the energy of the electron agrees with an eigenvalue of the QD array [12–14,19]. There are two reasons that explain the movement of these zeros when  $V_g$  varies. First, obviously,  $\epsilon_{t0}$  will be different from zero; so the role of  $\epsilon$  is now played by  $\epsilon - \epsilon_{t0}$ ; therefore the zeros will be at  $\epsilon_{t0} + \epsilon_{i0}$ . Secondly,  $t_{SO}^t$  will increase or decrease from its value when  $V_g = 0$  depending on the sign of  $V_g$ ; and it has been verified that if only  $t_{SO}^t$  is increased maintaining  $\epsilon_{t0} = 0$ , the zeros move slightly from their initial positions: the positive ones towards higher levels and the negative towards even more negative values. It is shown in Fig. 2 that the maxima of  $T_{\uparrow\uparrow}$  are not dependent of the Rashba parameter  $t_{SO}^t$  in the transversal chain; they depend only on the RSOI in the QW. In case of identical materials for the QW and the TC, the zeros of  $T_{\uparrow\uparrow}$  are also of  $T_{\uparrow\downarrow} (= T_{\downarrow\uparrow})$ ; actually,  $T_{\uparrow\downarrow}$  is a copy of  $T_{\uparrow\uparrow}$  but with a notably reduced value.

A major application of the setup of Fig. 1, is when the transversal chain is formed by a uniform magnetic material in which  $\epsilon_{t0\uparrow} = -\epsilon_{t0\downarrow} = -\epsilon_{t0}$ , and the QW is a normal conductor. Then,  $T_{\uparrow\uparrow} \neq T_{\downarrow\downarrow}$ . In Figs. 3 and 4 the QW is characterized by the same parameters as above; the coupling of the TC to the QW is specified by  $t_{y0} = 1.0$  and  $t_{SO}^c = 0.1$ ; and the TC considered as  $t_y = 1.0$ , NT = 3,  $t_{SO}^t = 0.1$ ,  $\epsilon_{t0} = 0.2$  in (a) and  $\epsilon_{t0} = 0.4$  in (b). For reference, the curve with  $\epsilon_{t0\uparrow} = \epsilon_{t0\downarrow} = 0$  is also displayed showing the three zeros of transmission and the two resonances in the energy band. Now  $T_{\uparrow\uparrow}(\epsilon) = T_{\downarrow\downarrow}(-\epsilon)$ , and at the Fano antiresonances we can get a perfect cancellation of spin-down transmission  $T_{\downarrow\downarrow} = 0$  (or  $T_{\uparrow\uparrow} = 0$ ), and a high spin-up transmission (and a high  $T_{\downarrow\downarrow}$ ). For instance, in Fig. 3a at  $\epsilon \approx -1.2$   $T_{\downarrow\downarrow} = 0$  and  $T_{\uparrow\uparrow}$  takes a maximum value. Also, the zero of  $T_{\uparrow\uparrow}$  at approximately  $-1.6$  is to the right of the maximum of  $T_{\downarrow\downarrow}$  at  $\sim -1.75$ . From Fig. 3b, it is shown that the zero of  $T_{\uparrow\uparrow}$  has moved to a more negative value while the maximum of  $T_{\downarrow\downarrow}$  has moved in the opposite direction; so, there must exist a value of  $\epsilon_{t0}$  in which the spin polarization of the transmission, given by  $P_T = (T_{\uparrow\uparrow} - T_{\downarrow\downarrow}) / (T_{\uparrow\uparrow} + T_{\downarrow\downarrow} + T_{\uparrow\downarrow} + T_{\downarrow\uparrow})$  [16,23], is very near to  $\pm 1.0$ . Results of this

kind are also found in [22] using a QD ring side-coupled to a QW, and in [18] with a two-level QD created by an applied magnetic field and coupled to ferromagnetic leads. In the negative interval of energy, when  $\epsilon_{t0}$  increases, the maxima of  $T_{\uparrow\uparrow}$  hardly reduce while those of  $T_{\downarrow\downarrow}$  decrease significantly; in the positive range of energies  $T_{\uparrow\uparrow}$  and  $T_{\downarrow\downarrow}$  change their roles. This reduction of  $T_{\downarrow\downarrow}$  and  $T_{\uparrow\uparrow}$  in different ranges of energy when the separation between the two levels rise, is related to the increase of  $T_{\uparrow\downarrow} (= T_{\downarrow\uparrow})$ , as shown in Fig. 4. Note that  $T_{\uparrow\downarrow}(\epsilon) = T_{\uparrow\downarrow}(-\epsilon)$ . Simulations with NT = 2, 4 and 5 were done. The higher NT is, there will be the more resonances and antiresonances, they will be narrower and; the scattering between the up and down levels in the TC will be higher. Therefore, with a fixed  $\epsilon_{t0}$ ,  $T_{\uparrow\downarrow}$  will be more remarkable. These results show that the device in Fig. 1 is a robust candidate to build a spin filter.

#### 4. Summary

In summary, the non-equilibrium Green's function method has been used to calculate the spin-dependent transmission of a QD array side-coupled to a QW which is attached to leads. RSOI through the QD array and the QW are considered, and the recursive Green's function method is used to reduce the whole system to a chain of three equivalent sites. When the QD array is made of a magnetic material with  $\epsilon_{t\uparrow} = -\epsilon_{t\downarrow} = -\epsilon_{t0}$ ; it is shown that there are values of  $\epsilon_{t0}$  in which the spin polarization transmission can be very near to 1.0; that is, this device is a candidate to be a spin filter.

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#### References

- [1] A.V. Moroz, C.H.W. Barnes, Phys. Rev. B 60 (1999) 14272.
- [2] F. Mireles, G. Kirczenow, Phys. Rev. B 64 (2001) 024426.

- [3] L.W. Molenkamp, G. Smidt, G.E.W. Bauer, *Phys. Rev. B* 64 (2001) 121202.
- [4] Q.-F. Sun, J. Wang, H. Guo, *Phys. Rev. B* 71 (2005) 165310.
- [5] B.K. Nikolić, S. Souma, *Phys. Rev. B* 71 (2005) 195328.
- [6] D. Sánchez, L.L. Serra, *Phys. Rev. B* 74 (2006) 153313.
- [7] D. Sánchez, L.L. Serra, M.-S. Choi, *Phys. Rev. B* 77 (2008) 035315.
- [8] F. Chi, J. Zheng, *Superlattice Microst* 43 (2008) 375.
- [9] M. Wimmer, M. Scheid, K. Richter, arXiv:cm0803.3705v1.
- [10] F.J. Jedema, et al., *Nature* 416 (2002) 713.
- [11] V. Sih, W.H. Lau, R.C. Myers, V.R. Horowitz, A.C. Gossard, D.D. Awschalom, *Phys. Rev. Lett.* 97 (2006) 096605.
- [12] E. Tekman, P.F. Bagwell, *Phys. Rev. B* 48 (1993) 2553.
- [13] P.A. Orellana, F. Domínguez-Adame, I. Gómez, M.L. Ladrón de Guevara, *Phys. Rev. B* 67 (2003) 085321.
- [14] M.L. Ladrón de Guevara, F. Claro, P.A. Orellana, *Phys. Rev. B* 67 (2003) 195335.
- [15] K. Kobayashi, H. Aikawa, A. Sano, S. Katsumoto, Y. Iye, *Phys. Rev. B* 70 (2004) 035319.
- [16] M. Popp, D. Frustaglia, K. Richter, *Nanotechnology* 14 (2003) 347.
- [17] A.A. Kiselev, K.W. Kim, *J. Appl. Phys.* 94 (2003) 4001.
- [18] P.A. Orellana, M. Amado, F. Domínguez-Adame, *Nanotechnology* 19 (2008) 195401.
- [19] W. Gong, Y. Zheng, Y. Liu, F.N. Kariuki, T. Lü, *Phys. Lett. A* 372 (2008) 2934.
- [20] F. Zhai, H.Q. Xu, *Phys. Rev. B* 76 (2007) 035306.
- [21] K. Shen, M.W. Wu, *Phys. Rev. B* 77 (2008) 193305.
- [22] M. Lee, C. Bruder, *Phys. Rev. B* 73 (2006) 085315.
- [23] S. Bellucci, P. Onorato, *J. Phys.: Condens. Matter* 19 (2007) 395020.
- [24] S. Datta, *Electronic Transport in Mesoscopic Systems*, Cambridge University Press, 1995.
- [25] H. Haug, A.-P. Jauho, *Quantum Kinetics in Transport and Optics of Semiconductors*, Springer Series in Solid-State Sciences, Vol. 123, Springer, 1996.
- [26] Z.Y. Zeng, Yi-You Nic, F. Claro, W. Yan, *Phys. Rev. B* 65 (2002) 193405.
- [27] R. Lake, G. Klimeck, R. Chris Bowen, D. Jovanovic, *J. Appl. Phys.* 81 (1997) 7845.
- [28] D.K. Ferry, S.M. Goodnick, *Transport in Nanostructures*, Cambridge University Press, 1997.